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COMPARISON OF THE VARIATION-THEORY AND END-POINT RESULTS FOR TAMPED SPHERES

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ABSTRACT

The end point and variation results presented in LA-53 and BM-144 (MS-97) agree throughout that part of the range of applicability of the end-point method (core and tamper thicknesses greater than 0.3 attenuation distances) in which the variation results have been calculated (up to 1.0 attenuation distance). For smaller dimensions the variation results are valid, and for larger sizes the end-point method applies. Large discrepancies are found only for the extrapolated portions of the variation results.

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COMPARISON OF THE VARIATION-THEORY AND END-POINT RESULTS FOR TAMPED SPHERES

In the treatment of tamped spheres having the same mean free path in the core and tamper two methods have been used extensively, variation theory and the end-point method.

The results of variation calculations are given in BM-144 (MS-97), THE CRITICAL RADIUS AND THE TIME CONSTANT OF A SPHERE IMBEDDED IN A SPHERICAL SCATTERING CONTAINER, B. Davison and K. Fuchs. Their results are presented graphically in a variety of forms, many of which are convenient for direct application. The primary data of these curves are restricted to core radii between 0.3 and 1.0 mean attenuation distances, and tamper thicknesses less than or equal to the core radius except for critical sizes with infinite perfect tampers. (The mean attenuation distance is $1/(N\sigma + \alpha/v) \cong 1/N\sigma(1+\gamma) = \lambda/(1+\gamma)$ where σ is the total transport cross section, N the number of nuclei per unit volume, v the neutron velocity, and α the increase of the natural logarithm of the neutron density per unit time.) In many of the graphs the limiting values at zero radius can be obtained from the bare-sphere results (which are accurate for all radii) thus permitting interpolation between radii of 0.0 and 0.3 mean attenuation distances. In the opposite limit, very large core and tamper thickness, diffusion theory becomes valid and permits a rough interpolation of some of the quantities presented for radii greater than 1.0. Those curves referring to finite core radii and infinite tampers are obtained by extrapolation from the finite-tamper results. This extrapolation can be guided by the diffusion-theory results only in their limit for large core radius. The extrapolation must therefore be regarded as unreliable except for critical conditions with perfect tampers.

The accuracy of the end-point method is limited by the same physical approximations used in the variation theory and in addition by the assumption that the

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boundaries involved are sufficiently far from each other. For large core diameter[1] and tamper thickness the end-point method may be presumed to give an accurate solution of the integral equation by which both methods approximate the physical problem of the tamped sphere. An early provisional limit of the range of sizes over which the method is accurate was given by a comparison of the bare-sphere results with variation theory. The difference is visible on a graph only for core diameters less than 0.3 mean attenuation distances. Since for the bare sphere only the simplest type of boundary is involved it should not be expected that equally accurate results would be obtained for tamper thicknesses slightly greater than 0.3. It was, however, expected that for core and tamper thicknesses between 0.3 and 1.0 the two methods should have some common range of validity.

The two sets of results were first compared for infinite tampers. For non-critical sizes large discrepancies were observed (amounting in some cases to a few tenths in γ/f). This discrepancy is ascribed to the extrapolation of the variation results. The two results were then compared for cases within the range of the finite-tamper primary variation theory data. (The primary data themselves are for the most part not available but are represented by analytic functions fitted to the primary data with a maximum error of 1% in $1+\gamma$.) The two methods agree in this range to within the accuracy with which the variation-theory results are presented, i.e., to about 1% in $1+\gamma$. The discrepancy at the lower end of this range, for tamper thickness of 0.3 mean attenuation distances, is about equal to this permitted error in the variation-theory results. It must, however, be ascribed to the inaccuracy of the end-point method since the variation method is most accurate at that end and the end-point results are approaching a limit for zero tamper thickness which is in error by several

1) The tamper thickness enters in the end-point theory in a way analogous to the core diameter rather than the core radius.

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percent in $1 + \gamma$. (This limit can be shown to differ appreciably from the bare sphere result common to the two methods.)

The end-point method may well be used, therefore, for all problems for which the core diameter and tamper thickness are appreciably greater than 0.3 mean attenuation distances. Its error will be quite negligible for large sizes and nowhere greater than 1% in $1 + \gamma$. For tamper thicknesses less than, say, 0.4 the variation results should be used and will be accurate to less than 1% down to zero thickness. For bare spheres of reasonable size the two methods agree and are accurate to a few hundredths of 1%. For radii less than .15 the variation results should be used. Both results are given in LA-53-A.

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